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PROCEEDINGS
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THE CONSTITUTION OF SECRETIN

By Carl Niemann

Gates and Crellin Laboratories of Chemistry, California Institute of Technology

Communicated May 13, 1939

The notable investigations of Ågren[1] have shown that the hormone secretin can be obtained in a homogeneous condition and that this hormone is a polypeptide, yielding upon degradation 3 mols of lysine, 2 mols each of arginine, proline and amide nitrogen and 1 mole each of histidine, glutamic acid, aspartic acid and sulfur.

In view of the fact that in his most recent communication[1a] Ågren has stated that the secretin molecule contains about 50 amino acid residues, it is of interest to note that a different conclusion can be reached on the basis of Ågren's experimental data.

Secretin chloride, dried *in vacuo* at 50°, is reported to contain 14.53 ± 0.18% nitrogen and 0.675 ± 0.005% sulfur.[1b] In a sample of 14.35% nitrogen it was found that 34.76% of the total nitrogen was basic nitrogen.[1b] From these values it follows that 100 gm. of secretin chloride contains 1.038 mols of nitrogen, 0.356 mols of basic nitrogen and 0.021 mols of sulfur. The ratio S:N = 1:49.2, the ratio S:basic N = 1:16.9 and the ratio basic N:N = 16.9:49.2. The molecular weight of secretin as determined by the ultracentrifugal method of Svedberg is reported to be approximately 5000[1] and as the minimum molecular weight calculated on the basis of 1 atom of sulfur per molecule of secretin is within 10% of this value we can draw the rigorous conclusion that one molecule of secretin contains 1 atom of sulfur and 49 atoms of nitrogen, 17 of which are precipitable by phosphotungstic acid under the conditions of the Van Slyke analysis.

A partition of the basic nitrogen fraction revealed [1b] that 100 gm. of secretin, dried as described above, contained 0.124 mols of lysine-nitrogen, 0.154 mols of arginine-nitrogen and 0.078 mols of histidine-nitrogen, or 0.062 mols of lysine, 0.039 mols of arginine and 0.026 mols of histidine. On examination of table 1 it is seen that the ratio of lysine:arginine:histidine

is 3:2:1, as was previously indicated by Ågren.[1] The agreement between found and calculated values is sufficiently close so as to preclude all other simple integral ratios. Therefore of the 17 nitrogen atoms in the basic fraction 3 atoms are present in the lysine side chains, 6 in the arginine side chains and 2 in the histidine side chain, a total of 11 nitrogen atoms in the side chains of the basic amino acids. Ågren has further reported[1b] that in a sample of secretin chloride containing 14.35% nitrogen, 4.65% of the total nitrogen appeared as amide nitrogen under the conditions of the Van Slyke analysis. It is well known that the Van Slyke procedure yields values for amide nitrogen which are at least 10% greater than the true

TABLE 1

COMPOSITION OF SECRETIN

AMINO ACID RESIDUE	GM. MOLS/100 GM. SECRETIN		RATIO (NUMBER OF INDIVIDUAL RESIDUES PER MOLECULE)	FREQUENCY[b] (RECIPROCAL FRACTION OF TOTAL NUMBER OF RESIDUES)
	CALCULATED[a]	FOUND		
Lysyl	0.063	0.062	3	12
Arginyl	0.042	0.039	2	18
Prolyl	0.042	0.042[c]	2	18
Histidyl	0.021	0.026	1	36
Glutaminyl	0.021	0.020[d]	1	36
Asparginyl	0.021	0.021[e]	1	36
Methionyl[f]	0.021	0.021	1	36

[a] Base = 0.021, i.e., (0.021 + 0.356/17)/2.

[b] On basis of total number of residues = 36.

[c] Mean of the non-amino nitrogen in filtrate from the basis, 0.050 and 0.033 as determined by isolation.

[d] Mean of 1/2 of the amide nitrogen, as determined by the Van Slyke method, 0.024 and 0.015 as determined by isolation.

[e] Mean of 1/2 of the amide nitrogen, 0.024 and 0.017 as determined by isolation.

[f] Sulfur assumed to be present as methionyl.

values and as the molar ratio of S:amide = 1:2.3 we may safely conclude that the secretin molecule contains 2 atoms of amide nitrogen. Therefore in addition to the 11 atoms of nitrogen present in the side chains of the basic amino acid residues we have 2 additional atoms of nitrogen present in the side chains of the dibasic amino acid residues as amide groups. Because of the apparent absence of any other amino acid residue containing nitrogen atoms in side chains[1] we may conclude that of the 49 atoms of nitrogen present in the secretin molecule 13 atoms are present in side chains and the remainder, i.e., 36 atoms of nitrogen, are present in the main peptide chain, and that the secretin molecule accordingly contains 36 amino acid residues. As Ågren has shown[1] that secretin contains no labile sulfur we may provisionally conclude that secretin contains 1 mol of methionyl and,

from the data presented in table 1, that the partial empirical formula of secretin is:

$$L_3Ar_2P_2H_1Gl_1As_1M_1X_{25}$$

where L = lysyl, Ar = arginyl, P = prolyl, H = histidyl, Gl = glutaminyl, As = asparaginyl, M = methionyl and X = the unknown residues.

The molecular weight of secretin calculated on the basis of a sulfur content of 0.675% is 4740. Ågren states[1] that secretin contains no free α-amino groups and it therefore follows that 1 mol of this cyclohexatriacontapeptide will yield, on hydrolysis, 36 amino acids weighing 5388 gm. The average weight of the 11 known amino acids is 145.3 and as the remaining 3790 gm. is distributed among 25 additional amino acids one might predict that their average weight is 151.6. However, it will be noted that the secretin employed in Ågren's investigations was dried *in vacuo* at 50° over P_2O_5. It is probable that these conditions are not sufficient to dehydrate the peptide completely even though a constant weight was obtained, and consequently the question of the average weight of the residues in secretin and the molecular weight of secretin must be deferred until additional experiments have been concluded. It is likely that the average weight of the unknown acids is less than 151.6 and the molecular weight of secretin less than 4740.

According to the periodicity hypothesis of protein structure[2] one would predict, from the ratios given in table 1, that the secretin molecule contains 12 amino acid residues or an integral multiple thereof, a result which is in perfect accord with the independent conclusion reached above. In addition it is seen that the secretin molecule conforms with the basic stoichiometrical principles of protein structure [2b,c,d] not only in respect to the numbers of the various individual amino acid residues but also in regard to the total number of residues present in the molecule.

[1] (*a*) G. Ågren, *Jour. Physiol.*, **94**, 553 (1938–1939); (*b*) *Idem, Skand. Archiv. Physiol.*, **70**, 10 (1934).

[2] (*a*) M. Bergmann, *Harvey Lectures*, **31**, 37 (1935–1936); (*b*) *Idem, Chem. Rev.*, **22**, 423 (1938); (*c*) M. Bergmann and C. Niemann, *Science*, **86**, 187 (1937); (*d*) *Idem, Jour. Biol. Chem.*, **115**, 77 (1936); **118**, 301 (1937); **122**, 577 (1938).

EOCENE AMYNODONTS FROM SOUTHERN CALIFORNIA

BY CHESTER STOCK

BALCH GRADUATE SCHOOL OF THE GEOLOGICAL SCIENCES, CALIFORNIA INSTITUTE OF TECHNOLOGY

Communicated April 15, 1939

Introduction.—Since the description[1] of a skull of *Amynodontopsis bodei* and of a lower jaw referred to this species from the uppermost Eocene, further dental parts have become available for study from localities in the Sespe. These furnish additional characters of value in recognizing the species represented in western Eocene deposits. Now also available is a fragment of skull with cheek-teeth from the Poway Eocene of San Diego County, California. This specimen possesses special significance because it permits a comparison with related forms in America and China. It is likewise of interest because it differs from the amynodonts of the Sespe and is clearly an earlier type.

Amynodon reedi, n. sp.

Type Specimen.—No. 2529, C. I. T. Vert. Pale. Coll., a fragment of the left side of the skull with $P\underline{4}$–$M\underline{3}$ inclusive, plate 1.

Locality.—Poway conglomerate and cross-bedded sandstones exposed at C. I. T. Vert. Pale. Loc. 314, San Diego, California.

Specific Characters.—Smaller than *Amynodon advenus*, *A. antiquus* and *A. erectus* and resembling *A. sinensis* in size. $P\underline{4}$ is narrow in anteroposterior diameter in comparison to its transverse width, more so than in *A. sinensis*. Principal axis of metaloph in $P\underline{4}$ at right angles to external border of tooth. I take pleasure in naming this species for Dr. Ralph D. Reed, penetrating student of California geology.

Description.—This species is distinctly smaller than *Amynodontopsis bodei* from the uppermost Eocene Sespe. It is likewise smaller than *Amynodon advenus* and related forms from the Uinta. Among American species nearest resemblance in size to *A. reedi* is found in a specimen, No. 1936A A. M. N. H. from the Uinta (Uinta B) of Utah. With regard to the latter it is interesting to record that its occurrence is the earliest among American amynodonts as at present known. No representatives of the group have thus far been uncovered in the Bridger Eocene. Presence of No. 1936A in Uinta B and its resemblance to No. 2529 from the Poway may be taken as evidence to indicate that the Poway is related in age to this stage of the Uinta.

Previously recorded mammalian species from the Poway suggest a faunal stage closely related to but slightly more advanced than the Bridger. Thus, *Yumanius woodringi* is a tarsiid primate more advanced than

Anaptomorphus. Presence of *Metarhinus*(?)*pater* suggests that the Poway is post-Bridger C and pre-Uinta C. The amynodont here described suggests again an age for the Poway comparable to that indicated by the titanothere and points more specifically to a time relationship with Uinta B. Unfortunately, the fauna thus far known from Uinta A is a meager one. It contains, according to Dr. H. E. Wood, 2nd., an amynodont which he recognizes as of the species *Amynodon advenus* (No. 11983 Carnegie Mus.). Presence of a larger species in the Uinta A horizon makes less certain a direct correlation between the Poway and the Uinta B on the basis of the amynodonts. A consideration of the Poway fauna as a whole may emphasize a time stage comparable to that of the lower Uinta or possibly with one situated between the lower Uinta and the upper Bridger.

COMPARATIVE MEASUREMENTS (IN MILLIMETERS)

	A. reedi, N. SP. NO. 2529 C. I. T. TYPE POWAY	*Amynodon*, SP. NO. 1936 A A. M. N. H. UINTA B	*A. sinensis*[1] TYPE CHINA	*A. antiquus* NO. 10047 PRIN. U. TYPE WASHAKIE
Length, anterior end of $P\underline{4}$ to posterior end $M\underline{3}$	103.8			
Length, anterior end of $M\underline{1}$ to posterior end $M\underline{3}$	85.8	86.2	73.5	93
$P\underline{4}$, anteroposterior diameter	*a*12		14.8	23.1
$P\underline{4}$, transverse diameter	24.7		22.1	
$M\underline{1}$, anteroposterior diameter	25.1	28.2	*a*24.5	36.4
$M\underline{1}$, transverse diameter	29.9	*a*31.5	25	
$M\underline{2}$, anteroposterior diameter	29.5	29.7	29	
$M\underline{2}$, transverse diameter	33.7	34.1	28.8	
$M\underline{3}$, anteroposterior diameter	27.9	32.4	25.1	
$M\underline{3}$, transverse diameter	31.1	34.2	27.8	

[1] Measurements after Zdansky.
a, Approximate.

Amynodon sp., possibly advenus Marsh

A single, imperfect tooth, $M\underline{2}$, No. 2537 C. I. T. Vert. Pale. Coll., Plate 2, figures 6, 6*a*, comes from locality 202 in the Sespe. This site is situated stratigraphically from 600 to 800 feet below locality 150 in the Sespe deposits exposed north of the Simi Valley, Ventura County, California, and represents an earlier (Upper Eocene) faunal horizon than that known from locality 150 (Uppermost Eocene).

No. 2537 approximates in size comparable teeth that have been referred to *Amynodon advenus*. However, teeth are present in the collections from locality 150 that are actually smaller than No. 2537. In contrast to one of these teeth, also an $M\underline{2}$, from locality 150, No. 2537 exhibits a smooth wall lining the valley between protoloph and metaloph.

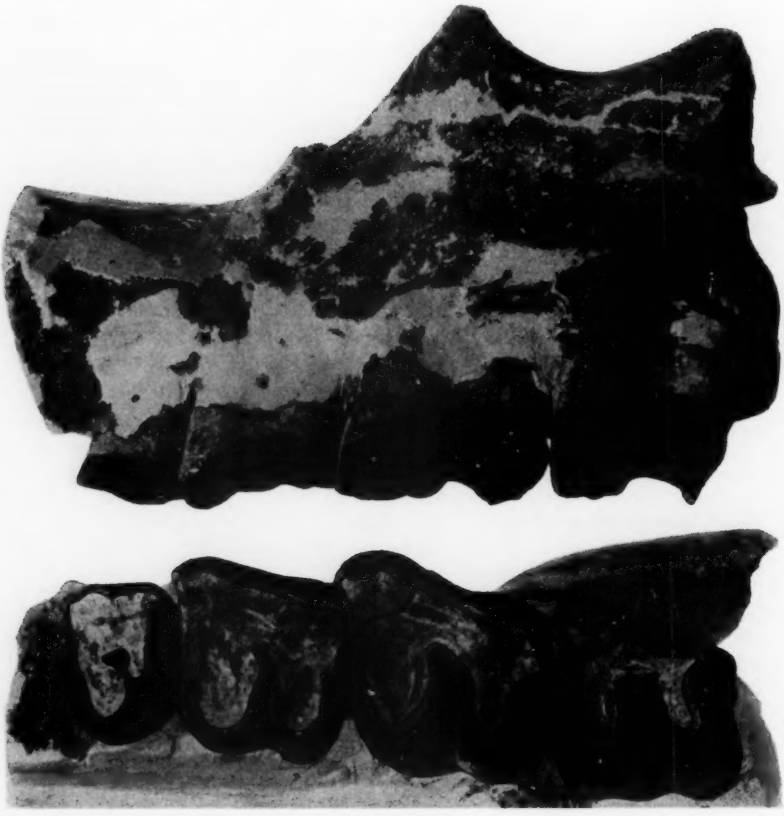


PLATE 1

Amynodon reedi, n. sp.

Type specimen, No. 2529 Calif. Inst. Tech. Coll. Vert. Pale., a fragment of skull with P4–M3, inclusive, lateral and occlusal views, × 1.

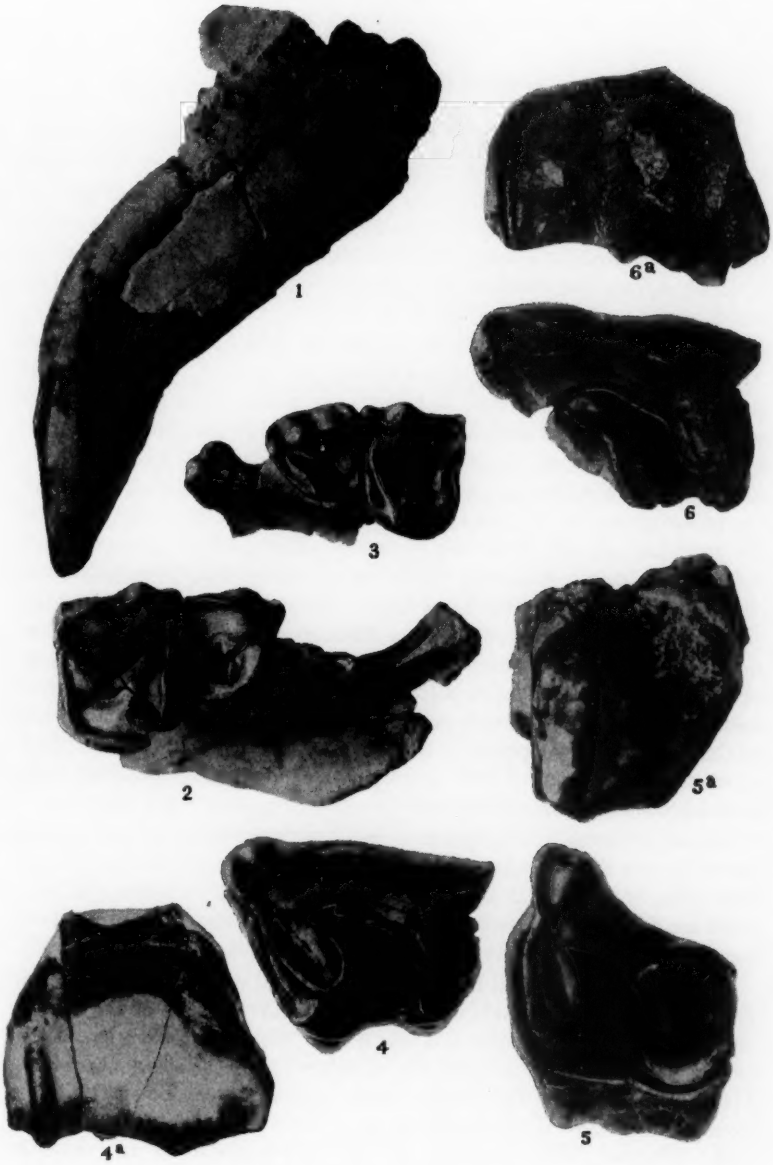
Poway Conglomerate, San Diego County, California. Upper Eocene.

PLATE 2 (RIGHT)

Figures 1 to 5a. *Amynodontopsis bodei* Stock. Figure 1, left canine, No. 2530, lateral view; figure 2, maxillary fragment with P3 and P4, No. 2532, occlusal view; figure 3, maxillary fragment with P2 and P3, No. 2531, occlusal view; figures 4, 4a, M2, No. 1089, occlusal and lateral views; figures 5, 5a, M3, No. 2535, occlusal and lateral views; × 1. Sespe Uppermost Eocene, California.

Figures 6, 6a. *Amynodon*, sp., possibly *advenus* Marsh. M2, No. 2537, occlusal and lateral views; × 1. Sespe Upper Eocene, California.

All specimens in Calif. Inst. Tech. Coll. Vert. Pale.


1
6a
6
3
2
5a
4
4a
5

Length of tooth, No. 2537, measured in millimeters along outer side, is 46.2.

Amynodontopsis bodei Stock

Referred to this genus and species are several parts of the dentition belonging to individuals smaller than the type, No. 1087 C. I. T. These specimens display additional characters not heretofore recorded for the Sespe amynodonts. The superior canine, No. 2530, shows a sinuous curvature in its longitudinal course as viewed from in front. The crown, which is broad in front and terminates in a sharp ridge behind, is approximately one-third the length of the tooth.

Premolars 1 to 3 are shown in specimens No. 2531, 2532, Plate 2, figures 3 and 2. In $P\underline{2}$ the crown is almost as wide as it is long. On the occlusal surface are seen the protoloph and metaloph, which spring from the inner wall of the ectoloph as narrow bands but become swollen along their inner halves. The protoloph is distinct from the anterior border of the tooth, but fuses with the inner border. The metaloph is distinctly smaller than the protoloph and in the stage of wear shown by No. 2531 is distinct from the inner and posterior border of the crown. Thus, both the pre- and postfossettes are open. The inner wall of the principal cusp of the ectoloph is directed inward and backward and helps to define the border of the fossette.

The occlusal pattern in $P\underline{3}$ may not differ greatly from that in $P\underline{2}$. The metaloph has a more posterior position than in $P\underline{2}$; the prefossette may be open or closed internally. A noteworthy feature is the small longitudinal folds which give added complexity to the forward side of the metaloph and to the inner wall of the prefossette. In $P\underline{4}$ the protoloph is well developed. A relatively thin, curved crest connects the rudimentary metaloph with protoloph, thus closing a prefossette internally.

Among upper molars collected at locality 150 in the Sespe are several similar in size to the molars in the type of *A. bodei*. At least two teeth, Nos. 1089, 2535, Plate 2, figures 4 and 5, are distinctly smaller. If these teeth are correctly assigned to *A. bodei*, they demonstrate the existence of appreciable variation in size among individuals of this amynodont.

$M\underline{2}$, No. 1089, Plate 2, figure 4, exhibits at least two wrinkles of the enamel lining the inner side of the ectoloph near the head of the median valley separating protoloph and metaloph. Similar modification of the enamel surface may be seen on the anterior side of the metaloph in $M\underline{3}$, Plate 2, figure 5. In this respect these teeth of the Sespe species differ from those of other Eocene amynodonts and are presumably more specialized. $M\underline{3}$, No. 2535, in contrast to the comparable tooth in *Amynodon reedi*, possesses a higher crown.

MEASUREMENTS (IN MILLIMETERS) OF TEETH OF *Amynodontopsis bodei*

Superior canine, No. 2530, anteroposterior diameter at base of crown	19.4; width 15.1
$P\underline{2}$, No. 2531, anteroposterior diameter on outer side	15.2; width 15.1
$P\underline{3}$, No. 2531, anteroposterior diameter on outer side	16.6; width 20.7
$P\underline{3}$, No. 2532, anteroposterior diameter on outer side	16.4; width 21.1
$P\underline{4}$, No. 2532, anteroposterior diameter on outer side	20 ; width 26.2
$M\underline{2}$, No. 1089, anteroposterior diameter on outer side	42.8; width 31.4
$M\underline{3}$, No. 2535, anteroposterior diameter on outer side	38.1; width 36.5

[1] Stock, C., *Proc. Nat. Acad. Sci.*, **19**, 762–767 (1933); **22**, 263–265 (1936).

THE ACTIVE UPTAKE OF IONS INTO CELLS AND ORGANISMS

BY AUGUST KROGH

LABORATORY OF ZOÖPHYSIOLOGY, UNIVERSITY OF COPENHAGEN

Read before the Academy, April 24, 1939

It is well known that the ionic composition of the protoplasm in animal cells differs significantly from that of the surrounding lymph or blood, potassium and phosphate being generally preponderant in the former and sodium and chloride in the latter. Since there is practically always osmotic equilibrium the ions within the cells must be largely free and in aqueous solution. Generally the actual concentrations are not well known, and only in the case of certain eggs has it been possible to show that certain ions are absorbed and concentrated from quite dilute outside solutions.

In the large plant cells studied by Osterhout, Collander[1] and others, active absorption of ions into the cell sap has been repeatedly observed and mechanisms, especially for the active absorption of potassium ions, have been suggested.

In plant roots Lundegårdh[1] has demonstrated an active transport of anions from very dilute external solutions into the much more concentrated sap rising in the stems. He finds that both monovalent and divalent anions can be so absorbed, but that the absorption requires a large expenditure of energy, provided probably by catabolism of carbohydrate. The CO_2 produced seems to act as part of the absorption machinery and the whole process is intimately bound up with the growth of the roots.

In a number of freshwater animals we have in my laboratory studied much more specialized mechanisms for absorbing and concentrating certain ions. These mechanisms come into play when the salt content of the body fluid has been depleted, e.g., by prolonged treatment with distilled water, and their biological significance is the maintenance of a more or less constant concentration in the body fluids round about a hundred times higher than that of the surrounding fresh water. In all the cases more

closely studied we have found separate mechanisms for the absorption of anions and cations, respectively. When a salt-depleted animal, like a frog, goldfish, crayfish or dragon-fly larva, is placed in a suitable volume (generally 2–5 times that of the animal) of millimolar NaCl it will take up both Na and Cl, but not necessarily at the same rate. In many cases the Cl concentration of an experimental solution is reduced to less than 0.005 mM. showing Cl to be taken up finally against a gradient of 1 to more than 10,000. Placed in NH_4Cl or $CaCl_2$ it will absorb only Cl, leaving out the cation, and transferred from this to NaCl it will now absorb Na in excess of Cl. Corresponding results are obtained when an experiment is started with $NaHCO_3$ or Na_2SO_4 from which only the cations are absorbed. After transference to NaCl an excess of Cl absorption will be shown in this case. This clearly shows the mutual independence of the two mechanisms. The absorption does not depend upon the osmotic inflow of water which takes place simultaneously, but can be demonstrated also from isotonic solutions of sugar or sulphate. There can be no doubt that the absorption requires the expenditure of energy since it is inhibited (in frogs) by cyanide, but attempts to measure the energy involved have failed so far, and I can only state that it cannot be very large and is certainly much smaller than the energy involved in absorption into plant roots.

The most interesting point is probably the specificity of the mechanisms for active ion absorption. When one of the animals named is placed in a mixture of NaCl and KCl or Na_2SO_4 and K_2SO_4 it will absorb only Na and leave the K untouched. Other alkali metals have not so far been tested, but Ca is generally left unabsorbed. The anions Cl and Br are both absorbed actively by fishes and frogs, but other anions are not. Some of them, like nitrate, thiocyanate and iodide will penetrate slowly by diffusion into a frog, while the integument and gills of the goldfish are almost impermeable.

In certain Crustacea the mechanisms for active absorption of cations and anions, while definitely separate, are less specific. The form most extensively studied by us is the "woolhanded" crab, *Eriocheir sinensis*, which penetrates from the sea far into fresh water. This animal has not acquired the high degree of impermeability characteristic for true freshwater animals and, unlike these, it cannot produce a dilute urine. The constant losses of salt thus incurred are made good through extremely powerful absorption mechanisms, which allow a crab of 50 g. to absorb not less than 8 mg. Cl/h from a millimolar solution. These mechanisms do not, however, distinguish between the cations Na and K and will take up a series of chemically related anions, namely Cl, Br, CNS, CNO and N_3. All these become concentrated in the blood far above their concentrations in experimental outside fluids. The first four are comparatively harmless, but azide ($N{\equiv}N{=}N{-}$) is very poisonous and experiments can be continued only over 15–20 minutes.

Experiments with the shore crab, *Carcinus maenas*, which can penetrate far into brackish water, but not actually live in fresh water, have shown essentially similar, but less powerful mechanisms, taking up at least K and Na and Cl, Br and CNS. There can be no doubt that the ion absorbing mechanisms have been independently developed in many different forms, and we have a rather primitive stage in such development in the two crabs just mentioned.

The cells doing the work have not been definitely located so far. In the frog they are situated in the skin. In the freshwater fishes special experiments have shown them to be located in the gills, and there is some evidence to show that they can be acted upon through the nervous system. In many arthropods special groups of cells readily take up silver from silver nitrate and there is some evidence, though scarcely conclusive, for identifying these with the cation-absorbing apparatus.

Machinery for selective and active ion absorption—active in the sense that energy is expended to move ions against a concentration gradient—is very widely distributed throughout the animal kingdom although poorly developed in strictly marine invertebrates. It is present in most kidneys and has recently been demonstrated by Visscher[1] in the intestinal wall of mammals. All such mechanisms are probably derived from properties inherent in living cells as such. They may and probably do differ in many important respects. What I wish to submit is that the most highly specialized types are most likely to furnish clues by which to find out how they accomplish the trick.

[1] A detailed bibliography is given in my book, *The Osmotic Regulation in Aquatic Animals*, Cambridge University Press, 1939.

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*MAMMARY TUMOR DEVELOPMENT IN MICE OVARIECTOMIZED AT BIRTH**

BY GEORGE WOOLLEY, ELIZABETH FEKETE AND C. C. LITTLE

ROSCOE B. JACKSON MEMORIAL LABORATORY, BAR HARBOR, MAINE

Communicated May 1, 1939

Lathrop and Loeb[1] observed that castration of female mice below the age of six months led to a marked decrease in mammary cancer incidence, and an increase in the cancer age. Castration at 3 to 4 months completely, or almost completely, prevented the appearance of carcinoma of the breast.[2] Murray[3, 4] found 17.1 per cent mammary gland tumors in 200 mice castrated 28 to 35 days of age, while 207 normal but non-breeding females gave an incidence of 11.1 per cent. Cori[5] found that castration of mice

between 15 and 22 days of age entirely prevented the occurrence of spontaneous adenocarcinoma of the breast while the control of breeding mice showed a tumor incidence of 78.5 per cent.

The present experiment was an attempt to demonstrate further the influence of ovarian hormonal factors on mammary tumor development. The Jackson Laboratory dba strain of mice, in which there is 50.4 per cent mammary tumors in virgin females, was used.[†,6]

Eighty-two mice were anesthetized by chilling and were completely ovariectomized within 24 hours after birth. Of these, 22 mice or 26.8 per cent developed mammary tumors at an average age of 522 days. In 102 control mice in which one ovary had been similarly removed, 40 animals or 39.2 per cent developed mammary tumors at an average age of 461 days.

Since there have been a number of reports indicating ovarian regeneration in mice[7] a careful check was made for the presence or absence of ovarian tissue in all animals at autopsy. All suspicious nodules were carefully sectioned and examined. Especially careful search was made for ovarian regeneration in all animals which had developed mammary tumors and in many of them histological examination of serial sections of all tissue near the former ovarian site proved the absence of all ovarian tissue.

The appearance of mammary tumors in animals in which the ovarian influence had been completely removed throughout their entire lifetime was unexpected, the general opinion being that prepuberty castration reduced the rate to a negligible one. This led to further search for the cause of the unusually high mammary tumor incidence.

At six months the uterus was thin, thread-like and showed marked atrophy. In those animals which were killed at a later age, the uterus was enlarged and hypertrophied. Observation of vaginal smears and of microscopic sections of vaginal epithelium showed estrous cycles to occur in several of the mice examined beyond the age of one year.

Whole mounts of mammary glands at six months of age showed little growth to have taken place. Whole mounts of the glands beyond one year of age showed extensive duct development, while the presence of end buds gave evidence of active growth.

It was noted that the adrenal glands in the completely ovariectomized animals were in all cases enlarged and had yellowish nodules which gave the gland an uneven contour. The degree of the abnormality increased with the age of the animal. Similar gross abnormalities were never found in normal animals, but have been observed in a few castrate male mice examined by us.

In almost all the completely ovariectomized animals both adrenals, parts of the mammary gland and uterus were sectioned and examined microscopically. Both adrenals of all these mice contained areas of nodular hyperplasia involving and partly replacing all three zones of the cortex.

The nodules were composed of large fat-containing, and often pigmented, cells which in arrangement resembled the structure of the zona glomerulosa. These areas were not encapsulated. In some of the older animals, the medulla was pushed to the opposite side, due to the large size of the nodule. In three mice malignant tumors of the adrenal cortex were found.

The mammary glands of the older ovariectomized animals showed well-developed and extensive duct systems. In many, groups of alveoli were also present. In several mice the ducts and alveoli contained secretion. Most of the mammary gland tumors were adenocarcinomas, while a few could be more properly described as carcinoma simplex.

The uterus of the older animals contained dilated and in some cases cystic, endometrial glands. A very characteristic occurrence was a peculiar hyaline change of the connective tissue fibers of the subepithelial uterine mucosa.

Conclusions.—The possibility is suggested that: 1. The removal of both ovaries at 24 hours after birth allowed, in some manner, the nodular hyperplasia of the adrenal cortex; 2. The prolonged action of the hyperplastic adrenal probably stimulated the development of the mammary glands and the uterus in the older animals, thereby simulating or replacing ovarian activity.

Further studies are in progress.

* Supported in part by a grant from the Josiah Macy, Jr., Foundation.

† This, however, probably should not be considered an absolutely accurate figure for the stock at the present time as tumor incidence different from that reported by Murray has recently been recorded in some substrains of this stock.

[1] Lathrop, A. E. C., and Loeb, L., *Jour. Cancer Res.*, **1**, 1 (1916).
[2] Loeb, L., *Jour. Med. Res.*, **40**, 477 (1919).
[3] Murray, W. S., *Science*, **46**, 600 (1927).
[4] Murray, W. S., *Jour. Cancer Res.*, **12**, 18 (1928).
[5] Cori, C. F., *Jour. Exper. Med.*, **45**, 983 (1927).
[6] Murray, W. S., and Little, C. C., *Genetics*, **20**, 446 (1935).
[7] For a review see Pencharz, *Jour. Exper. Zoöl.*, **54**, 319 (1929).

OBSERVATIONS ON THE MECHANISM OF INDUCED CHROMOSOME REARRANGEMENTS IN SCIARA

BY C. W. METZ AND R. D. BOCHE

DEPARTMENT OF EMBRYOLOGY, CARNEGIE INSTITUTION OF WASHINGTON

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For several years efforts were made in this laboratory to induce mutations in the fungus fly, Sciara, by means of irradiation. These extensive studies, largely carried out by Dr. Helen Smith-Stocking,[1] seemed to show that the chromosomes of Sciara are peculiarly resistant to irradiation, as judged by the extremely small numbers of mutants secured and the high dosages tolerated. No adequate explanation could be given for such results, since both x-rays and radium were used and there was no reason to suppose that the rays failed to reach the chromosomes of the germ cells. In this work the flies irradiated were mostly females. Few males were treated, partly because of their poorer viability, but especially because in Sciara the chromosomes of males are not transmitted by their sons. The assumption was made from analogy with Drosophila,[2] that treating females would be as effective as treating males.

Since it is well known from the work of Muller,[3] Stadler[4] and subsequent investigators, that in general a correlation exists between induction of mutations and induction of gross chromosome rearrangements, it was postulated[5] that in Sciara rearrangements, like mutations, would show a low frequency. Such an expectation was heightened by the lack of gross chromosome rearrangements in hybrids between two species of Sciara.[6]

In order to get definite evidence regarding induced chromosome rearrangement we have recently carried out the experiments summarized below. Although these only represent the initial stages of the investigation they appear to warrant brief consideration at this time. Attention is confined here primarily to three topics: (1) The evidence, which indicates that chromosome rearrangements, and presumably mutations, may be secured readily by irradiating adult males (sperms), but not by irradiating adult females (unfertilized eggs). (2) The possible significance of the results in terms of the physical characteristics of the chromosomes. (3) The possible bearing of the phenomena on the question as to the relation between "gene mutation" and visible chromosome rearrangement. The latter two topics can only be treated very briefly, and with little discussion of other contributions to the subject.

We are greatly indebted to Dr. Louis R. Maxwell, of the Bureau of Chemistry and Soils, U. S. Dept. of Agriculture, for the x-ray treatments, and to the staff of the Kelly Hospital, Baltimore, for the radium treatments considered here.

Chromosome Rearrangements.—In the studies on induction of visible rearrangements only x-rays were used. All treatments were made with the same apparatus and under as nearly uniform conditions as possible, using an air-cooled Coolidge tube with tungsten anode under a constant potential of 45 k. v., with no filter. The dosage is considered to be accurate within 5%. In one experiment the males and females were irradiated simultaneously.

In all cases the determinations were made through study of the salivary gland chromosomes of F_1 larvae, and all by one observer (Metz). Both male and female larvae were used. With the one exception noted below all cultures are from pair matings.

1. *Treated Males.*—Observations on treated males were extended only far enough to show the approximate frequency of rearrangements and the presence of rearrangements in numerous cultures. Four experiments involved a dosage of 5000 r units. From these 17 cultures were tested. All revealed rearrangements except four, from each of which less than seven specimens were examined. All are included in the following summary:

(*a*) *Sciara ocellaris* Comst. 18 larvae; 10 unaffected; 6 affected, 2 unfavorable for study; 7 rearrangements. (*b*) *Sciara coprophila* Lint. 48 larvae; 18 unaffected, 12 probably unaffected; 6 (+3 ?) affected, 10 unfavorable; 6 rearrangements (plus 3 doubtful cases ?). (*c*) Same species; mass culture. 25 larvae; 6 (+4 ?) unaffected; 2 affected; 13 unfavorable; 2 rearrangements. (*d*) *Sciara reynoldsi* Metz, 72 larvae; 42 (+5 ?) unaffected; 12 (+1 ?) affected; 12 unfavorable; 13 (+3 ?) rearrangements.

Total: Counting only the clearly unaffected and the clearly affected cases, there is a total of 28 rearrangements (14 translocations and 14 inversions) in 104 larvae, or approximately 27%. If doubtful cases are included the percentage ranges between 22 and 27 depending on the basis of classification. In addition, other males were given 3000 and 7000 r units, with correspondingly different results.

2. *Treated Virgin Females.*—Females were all given 5000 r units. Three experiments were carried out, and material was examined from 13 cultures. In order to avoid selection all, or practically all, larvae in the culture were used in some cases. *S. ocellaris:* 65 larvae; 52 unaffected; 13 unfavorable for study. *S. reynoldsi:* Lot 1; 96 larvae; 88 unaffected; 8 unfavorable. Lot 2; 55 larvae; 45 unaffected; 10 unfavorable.

Total: 185 unaffected; 31 unsatisfactory for study. Ten of those classed as unaffected were not entirely satisfactory for study but were apparently unaffected.

No evidence of rearrangement was found in any material from treated virgin females. The only suggestion of such an effect in our results is the finding of three rearrangements in a preliminary experiment on *S. coprophila* in which females not known to be virgin were treated.

These results seem contrary to those secured in Drosophila, where no such extreme difference has been found between the response of germ cells in the two sexes.[2] The difference, however, is probably not concerned with sex, but with other factors. The clue to the solution presumably lies in the fact that in Sciara, unlike Drosophila, the eggs of any female all mature at once and are all nearly mature when she emerges. Hence only mature, or nearly mature, eggs have been irradiated, whereas in Drosophila oöcytes and oögonia in various stages are all irradiated simultaneously.

Although the number of gross rearrangements (or chromosome breaks) recorded here from treated Sciara males is probably significantly lower than that secured in Drosophila,[7] it shows clearly that such rearrangements are readily secured. Their absence in material from treated virgin females, therefore, indicates either that rearrangements are not produced with anything like the same frequency in the eggs used, or else that in some way the altered chromosomes or eggs are eliminated. We have been unable to find any evidence that the latter alternative is correct. Fertility does not appear to be reduced more by treating females than by treating males. On the contrary, females will tolerate treatments much more severe than those which appear to completely sterilize the males. Apparently no weeding out of modified chromosomes occurs unless it is by selective segregation at meiosis in the treated egg. This feature is being investigated, but the possibility of accounting for the results on such a basis seems remote. The one translocation manifest by a mutant character ("Stop") in Sciara is readily transmitted through both eggs and sperms.

Assuming a Poisson distribution of breaks in eggs and sperms, calculations indicate that we should have recovered between 24 and 40 rearrangements. While these calculations involve numerous assumptions they possess the advantage of giving an estimate of the minimum frequency of rearrangements expected from x-rayed eggs. So far as we know this matter has not been studied in Drosophila, although the same reasoning is applicable.

It is believed that the present results from treated females reflect an actual absence or very low incidence of rearrangements. This agrees with the genetic evidence mentioned above, concerning induced mutation from treated virgin females. What is lacking at present is adequate genetic evidence from treated males (sperms). One experiment (Smith-Stocking, unpublished), however, suggests that mutation may readily be induced by treating sperms. Here a mixture of males and fertilized as well as unfertilized females (of *S. reynoldsi*) was treated with radium. Among 1073 offspring following a 5 gram-hour treatment 2 mutants were found. From a treatment of 3 gram-hours one was secured among 281 individuals. Several other possible mutants appeared, but they were sterile.

Physical Characteristics of the Chromosomes.—It is considered probable that the difference between the frequency of rearrangement in sperms and in eggs is due to a difference in the physical characteristics of the chromosomes. Little is known regarding chromosomes in the sperm except that they are packed relatively closely together, and are presumably not in a condensed condition comparable to that obtaining during mitosis. The close proximity of the threads should give excellent opportunity for broken ends, caused by irradiation, to unite in new combinations and produce rearrangements.

In the eggs, during the time under consideration, the non-homologous chromosomes seem clearly not to be in such close proximity as in the sperms. Some females were treated on the first, others on the second, day after emergence; all with the same result. When the eggs become fully mature, on the second or third day, the chromosomes are apparently always in metaphase or anaphase of the first meiotic division, and fully condensed. During the first day, however, they are difficult to stain, and we have not yet determined just what condition they are in. Preliminary observations suggest that each tetrad is in the form of a well defined, but somewhat diffuse, prochromosome-like body, the matrix of which is a relatively firm gel. If this is the case the gel may be sufficiently resistant to prevent broken ends of chromosomes from moving enough to form new combinations. Synaptic association, of course, may act similarly.

It seems not improbable, as suggested in an earlier paper,[8] that irradiation serves to lessen the viscosity of the matrix or sheath material, which normally insulates the chromosome threads, and that this factor, together with proximity and perhaps activity of movement, serves to determine the frequency with which rearrangements are induced. Such an effect may be localized at the regions of chromosome breakage and due to the same causes as the breaks. There is considerable evidence to indicate that both the amount and the viscosity of the matrix-sheath material differ noticeably in different cells and under different conditions. This might serve to account for such observed differences in susceptibility as those between dormant and germinating seeds, mentioned above, between germ cells in different stages of development,[9] and between young and old sperms,[10] as well as the well-known differences between cells in different stages of the cell cycle.[11]

The possible bearing of the present phenomena on the relation between chromosome rearrangement and "gene mutation" is under investigation. It need only be observed here that if both effects may be produced readily by treating sperms, but not by treating eggs, as the evidence suggests, the question is again raised as to whether there is any essential difference between the initial process in "gene mutation" and that which gives rise to chromosome rearrangement.

[1] Smith, H. B., *Proc. 6 Int. Cong. Genetics*, **2**, 187 (1932); Smith-Stocking, Helen, *Genetics*, **21**, 421 (1936); Metz, C. W., *Carnegie Inst. Wash. Pub. 501*, **275** (1938).

[2] See, e.g., Timofeef-Ressovsky, *Mutationsforschung in der Vererbungslehre*, pp. 60, 61, Steinkopf, 1937.

[3] Muller, H. J., *Proc. 6 Int. Cong. Genetics*, **1**, 213 (1932).

[4] Stadler L. J., *Ibid.*, **1**, 274 (1932).

[5] Metz, C. W. (*loc. cit.*), and Metz, C. W., *Genetics*, **24**, 105 (1939).

[6] Metz, C. W., and Lawrence, E. G., *Jour. Hered.*, **29**, 179 (1938).

[7] In terms of chromosome breaks we find approximately 50% in Sciara compared with 102 and 125% in Drosophila. Data for Drosophila from Catcheside, D. G., *Jour. Genetics*, **36**, 307 (1938) and Bauer, H., Demerec, M., and Kaufmann, B. P., *Genetics*, **23**, 610 (1938).

[8] Metz, C. W., these PROCEEDINGS, **20**, 159 (1934).

[9] Shapiro, N. J., *Biol. Zhur.*, **6**, 837 (1937).

[10] Offermann, C. A., *Genetics*, **24**, 81 (1939).

[11] See, e.g., Sax, K., *Genetics*, **23**, 494 (1938).

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*EVIDENCE FOR THE EXISTENCE OF AN ELECTRO-DYNAMIC FIELD IN LIVING ORGANISMS**

BY H. S. BURR AND F. S. C. NORTHROP

YALE UNIVERSITY

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There are two major classical theories of modern science: particle physics and field physics. The chief difference between them was clearly stated by Clerk Maxwell in his initial paper on electro-magnetic theory. Particle physics, he writes, considers any phenomenon "as due to the mutual action of particles," "but we are proceeding on a different principle, and searching for the explanation of the phenomena, not in the currents alone but also in the surrounding medium"[1] or, to use the language of his third paper, "in the form of the relations of the motion of the parts."[2] In short, particle physics directs attention to the constituent particles, whereas field physics centers theory and experimentation upon the medium in which the system as a whole is imbedded and upon its structure.

Since the fundamental problem of biology is organization, it would appear that field physics is the more appropriate for its investigation. It was considerations similar to these, together with certain facts in experimental embryology,[3] which caused the writers in 1935 to propose the "electrodynamic theory of life."[4] It was this theory in turn which guided Burr, Lane and Nims to the construction of the vacuum-tube microvoltmeter,[5] and which suggested the experimental investigations [6–18] and findings which it is the purpose of the remainder of this paper to summarize.

In biology, the complexity of the living system is so great that investi-

gators have been content since the time of Aristotle to analyze and describe, in as much detail as possible, the nature of its component parts. However, every biologist knows that one of the most important problems is to describe quantitatively as well as qualitatively the relationships which are known to exist within cells and between them and their environment. To study this relatedness in the laboratory required that certain conditions be met. As pointed out by Lund in his brilliant and meticulous study of the bio-electric properties of the growing onion root tip, determination of the electrical properties of the living system can be carried out most advantageously by means of instruments which record only voltage differences independent of current and resistance. This condition precludes the use of nearly all standard instruments, save the quadrant electrometer or similar devices.

Using a vacuum-tube microvoltmeter with a high degree of sensitivity and excellent stability, it has been possible to explore the electrical properties of a wide variety of living forms undisturbed by changes in resistance in the organism and without disturbing in any significant way the inherent electrical properties of the thing measured.

This has been accomplished by designing an instrument which is essentially a vacuum-tube bridge. The input impedance is sufficiently high to draw a minimum of current from the system under measurement and is, therefore, independent of resistance changes in the measured system. Contact between the instrument and the living organism is made through silver-silver chloride electrodes immersed in physiological salt solution. Since these electrodes are non-polarizing and reversible, electrode artefacts are reduced to a minimum. The entire apparatus is shielded and grounded at appropriate points so that it can be said with a fair degree of certainty that the recorded deflections of the galvanometer spot give an accurate picture of the voltage differences in the living system.

Studies of the past five years have shown that in many vertebrates, as well as in plants and invertebrates, there is a relatively steady state voltage difference between any two points. These gradients are remarkably stable, are of considerable magnitude and are changed only by alterations in the fundamental biology of the organism. Moreover, in all the forms studied, the gradients are not chaotic but exist in a well-defined pattern which is characteristic of the species to which the animal belongs and is, to some extent, characteristic of the individual. In general, it may be said that growth and development, local injuries, the menstrual cycle and ovulation in the female, and the incidence of cancer profoundly affect voltage differences in what seem to be a unique manner.

Interesting as the above observations may be, it is more important to determine if the experimentally measured pattern of voltage differences determines in any fundamental sense the organization of the system. In

this connection the physical chemist, Teorell,[19] has made an important observation. He has shown that a physical system undergoing chemical reactions produces active ions which pass through a membrane with different mobilities. This results in an electric field whose forces determine the distribution and motion of all the passive ions in the system. Thus, his theory, derived from Ostwald, Nernst and Planck, provides meaning both for the determination of the electric field by the active ions and the determination of the position and motion of the passive ions by the electric field. This, it is to be noted, is a special case that is in accord with the fundamental thesis of the electro-dynamic theory of life. It was said that, "the pattern or organization of any biological system is established by a complex electro-dynamic field, which is in part determined by its atomic physico-chemical components and which in part determines the behavior and orientation of those components."

It is not surprising, therefore, that voltage gradients between the head and tail of Amblystoma and chick embryos can be determined with considerable certainty not only when contact is made directly with the surface of the organism, but also when the electrodes are from $1^1/_2$ to 2 mm. away from the surface of the embryo. It will be seen at once that this is a striking confirmation of the field concept since the distribution of the chlorine ions in the physiological salt solution surrounding the embryo is determined by the field of the organism in such a manner that it is capable of being measured by a vacuum-tube microvoltmeter.

A further striking bit of confirmatory evidence is found in the phenomenon which may be observed in the salamander embryo when it is revolving between the tips of a pair of capillary electrodes as a result of ciliary action. As the head passes first under one electrode and then under the other, corresponding oscillations of the galvanometer occur. Under these conditions the embryo is acting as an AC generator of very low frequency. Such a phenomenon can only be explained, so far as it is possible to see at present, on the assumption that an electrical field exists in the embryo.

In the course of many hundreds of thousands of determinations of voltage differences in women, it has been shown that a very definite bio-electric correlate of the menstrual cycle exists. It may be said with a fair degree of assurance that usually once but sometimes twice in the menstrual cycle there is a sharp rise in voltage difference which lasts for approximately 24 hours. Furthermore, it can be shown that this rise is in all probability associated with ovulation. The findings make it reasonably certain that these bio-electric correlates of ovulation may occur at any time in the cycle. Moreover, there is no tendency for the appearance of a rhythm in the time of ovulation. In fact, cycles occasionally appear without a bio-electric ovulatory peak. Hence it may be said that it is impossible to predict the time of ovulation. All that can be done is to record the time when it oc-

curs. These determinations have been made between the right and left index fingers. They, therefore, reflect changes in physiological activity which are concentrated in the main in the generative tract. However, these changes are so profound as to produce undoubted alterations in the field of the whole organism. Explanation of the phenomenon on the basis of ionic transport or changes in the local chemistry is difficult. The phenomenon is relatively easy to understand, however, if an electro-dynamic field of the whole organism is assumed, the over-all properties of which may be altered by changes in local constituent factors.

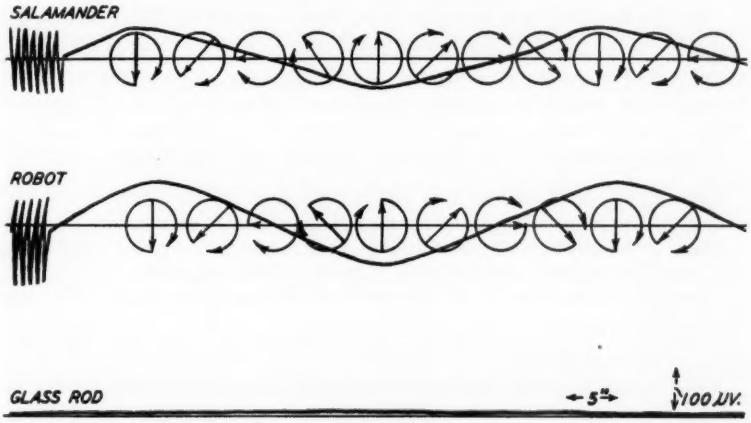

FIGURE 1

A tracing of three graphs from a recording galvanometer and a microvoltmeter demonstrating the similarity between the electrical field in a salamander and a copper-solder robot when each is rotated. Below, the complete absence of any such phenomenon when the glass rod is rotated.

In the same way, registration of the exact time of ovulation in the rabbit and in man, as confirmed by Reboul and Davis[20] and Rock,[21] may be assumed to be a field phenomenon, inasmuch as the measurements were made not directly on the ovary but between the symphysis and the vagina. Here it has been shown as a result of direct observation, that the bio-electric correlate of ovulation is coincident with the liberation of the ovum from the follicle, even though pickup electrodes are centimeters distant.

During an extensive study of the bio-electric correlates of cancer in mice, it was noted that in young mice a marked rise in voltage gradients across the chest occurred from two to three weeks before the new growth could be detected by palpation. This rise was independent of the locus in the organism of the new growth. Apparently rapidly growing masses of cells,

too small for direct observation, produce local bio-electric changes in the organism which effect the field properties of the whole system in such a manner as to make it possible to record the changed voltage gradients across the chest.

It seems clear from the above that an understanding of a wide variety of bio-electric phenomena in the living organism can best be reached by the assumption of an electro-dynamic field in the organism.

* Aided by a grant from the Josiah Macy, Jr., Foundation.

[1] Maxwell, Clerk, *Scientific Papers*, **1**, 193 (1890).

[2] Maxwell, Clerk, *Ibid.*, **1**, 533 (1890).

[3] Burr, H. S., *Jour. Comp. Neur.*, **52**, No. 2, 347–371, Dec. 15 (1932).

[4] Burr, H. S., and Northrop, F. S. C., *Quart. Rev. Biol.*, **10**, No. 3, 322–333, Sept. (1935).

[5] Burr, H. S., and Lane, C. T., *Yale Jour. Biol. Med.*, **8**, No. 1, 31–35, Oct. (1935).

[6] Burr, H. S., Hill, R. T., and Allen, Edgar, *Proc. Soc. Exptl. Biol. Med.*, **33**, 109–111 (1935).

[7] Burr, H. S., Lane, C. T., and Nims, L. F., *Yale Jour. Biol. Med.*, **9**, No. 1, 65–76, Oct. (1936).

[8] Burr, H. S., and Musselman, L. K., *Ibid.*, **9**, No. 2, 155–158, Dec. (1936).

[9] Burr, H. S., and Northrop, F. S. C., *Growth*, **1**, No. 2, 78–88, Apr. (1937).

[10] Burr, H. S., and Hovland, C. I., *Yale Jour. Biol. Med.*, **9**, No. 3, 247–258, Jan. (1937).

[11] Burr, H. S., and Hovland, C. I., *Ibid.*, **9**, No. 6, 541–549, July (1937).

[12] Burr, H. S., Musselman, L. K., Barton, D., Kelly, N., *Science*, **86**, No. 2231, 312, Oct. 1 (1937).

[13] Burr, H. S., Musselman, L. K., Barton, D., and Kelly, N., *Yale Jour. Biol. Med.*, **10**, No. 2, 156–160, Dec. (1937).

[14] Burr, H. S., and Barton, Dorothy, *Ibid.*, **10**, No. 3, 271–274, Jan. (1938).

[15] Burr, H. S., and Musselman, L. K., *Am. Jour. Obst. Gynec.*, **35**, No. 5, 743–751, May (1938).

[16] Burr, H. S., Strong, L. C., and Smith, G. M., *Yale Jour. Biol. Med.*, **10**, No. 6, 539–544, July (1938).

[17] Burr, H. S., and Smith, Paul K., *Ibid.*, **11**, No. 2, 137–140, Dec. (1938).

[18] Burr, H. S., Harvey, S. C., and Taffel, Max, *Ibid.*, **11**, No. 2, 104–107, Dec. (1938).

[19] Teorell, T., *Proc. Nat. Acad. Sci.*, **21**, 152 (1935).

[20] Reboul, J., Davis, H., and Friedgood, H. B., *Am. Jour. Physiol.*, **120**, 724–732 (1937).

[21] Rock, J., Reboul, J , and Snodgrass, J. M., *Am. Jour. Obst. Gynec.*, **36**, 733–745 (1938).

CLASS FIELD THEORY OF SOLVABLE ALGEBRAIC NUMBER FIELDS

BY D. M. DRIBIN

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF NEBRASKA

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1. The class field theory, as it exists at the present time, is a theory of abelian algebraic number fields $K|k$ and the definition of a class field is, by its very nature—given in relation to a group of ideals in k—abelian. To extend such a theory to normal non-abelian fields it is necessary, first of all, to divorce the definition from abelian concepts. Again, if certain features of the ordinary class field theory are to be preserved, certain notions must be discarded or, at the very least, modified. Thus, in order to have an isomorphism theorem we shall do away with the ideal class group in $K|k$ but instead shall employ (in case $K|k$ is solvable) the various class groups in certain subfields $K_i|K_{i-1}$, where $K \supset K_i \supset K_{i-1} \supset k$ and where $K_i|K_{i-1}$ is abelian.

If $K|k$ is an algebraic extension of finite degree, we denote by $\Pi(K|k)$ the totality of prime ideals in k which have in K at least one prime ideal divisor of relative degree one.

DEFINITION: *Let* Π *be a set of prime ideals of* k *such that* Π *has a density* $d(\Pi) = 1/n$, *where* n *is a positive rational integer. If there exists a field* K|k *of degree* n *such that* $\Pi \subset \Pi(K|k)$ *and* $\Pi(K|k) - \Pi$ *has density zero, then* K|k *will be called a class field over* k *relative to* Π.

We shall call a set Π *admissible* if $d(\Pi) = 1/n$, where n is a positive rational integer. As a consequence of a theorem of Kronecker, it follows that if K is a class field corresponding to an admissible set Π, then $K|k$ is normal of degree n.

We then have the following theorems:

THEOREM (*Converse Theorem*). *A normal field* K|k *of degree* n *determines an admissible set* Π *of density* 1/n *and this set is uniquely determined except for a set of ideals of zero density.*

THEOREM (*Ordering Theorem*). *If* Π_1 *and* Π_2 *are two admissible sets of prime ideals such that* $\Pi_1 \supseteq \Pi_2$ *with the exception of a set of ideals of zero density, and if* K_1 *and* K_2 *are class fields over* k *with respect to* Π_1 *and* Π_2, *respectively, then* $K_1 \subseteq K_2$.

COROLLARY (*Uniqueness Theorem*). *If* Π_1 *and* Π_2 *satisfy the hypotheses of the theorem and if* $\Pi_1 = \Pi_2$ *with the exception of a set of ideals of zero density then* $K_1 = K_2$.

In the abelian class field theory the following theorem is demonstrated: $H_1 \cup H_2 \longleftrightarrow K_1K_2$, $H_1H_2 \longleftrightarrow K_1 \cup K_2$, where H_1 and H_2 are ideal groups and $K_1 \longleftrightarrow H_1$, $K_2 \longleftrightarrow H_2$ by the class field theory. The corresponding

theorem cannot be proved in the present situation, for if Π_1 and Π_2 are admissible sets of ideals, $\Pi_1 \cup \Pi_2$ and $\Pi_1 \Pi_2$ are not in general admissible.

2. We shall assume in what follows that $K|k$ is a normal field whose group is solvable. Then $K = K_t$ has a chain of subfields over k: $K_t \supset K_{t-1} \supset K_{t-2} \supset \ldots \supset K_1 \supset k$, which has the property that $K_i|K_{i-1}$ is abelian and that K_i is a maximal abelian subfield of $K_t|K_{i-1}$.

We shall say that Π is *immersed* in an ideal group H in k if Π is a subset of $\Pi(H)$, the set of prime ideals occurring in H. By means of the abelian class field theory the following theorem can be proved:

THEOREM (*Existence Theorem*). *Let* Π *be an admissible set of ideals in* k. *Then* Π *determines a solvable class field* K|k *if and only if* Π *is immersible in an ideal group* H *in* k *which has no proper ideal subgroup.*

An analogue can also be obtained of the Artin law of reciprocity in algebraic number fields. This generalized law yields a one-one multiplicative correspondence between the sets of conjugate automorphisms of $K|k$ and certain unordered vectors Γ whose components are ordered vectors of elements of the class groups defining the $K_i|K_{i-1}$. Incidentally, a non-abelian definition is given for multiplication of ideals in algebraic number fields.

ON THE REPRESENTATION OF GROUPS OF FINITE ORDER

BY RICHARD BRAUER

UNIVERSITY OF TORONTO

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1. *Introduction.*—The modular representations of a group $\mathfrak{G}$ of finite order have been studied by C. Nesbitt and the author in a joint paper[1] (cf. §2). These investigations will be continued in §3 of this note. The results enable us to derive a number of new properties of the ordinary group characters of $\mathfrak{G}$ in the case that g is of the form $g = pg'$ where p is an odd prime which does not divide g' (§4). I mention here some applications of this theory.

I. *Let* $\mathfrak{T}$ *be an irreducible group of linear transformations of degree* n *which has no normal subgroup of order* p. *If the order of* $\mathfrak{T}$ *is divisible by the prime* p *to the first power only, then* $p \leqq 2n + 1$. This improves for the case $g \not\equiv 0 \pmod{p^2}$ a theorem of H. F. Blichfeldt[2] who proved $p \leqq (2n + 1)(n - 1)$. For $p = 2n + 1$ we can show that $\mathfrak{T}$, considered as collineation group, is isomorphic with $LF(2, p)$.

II. *If a group of order* $g = p.q.r^m$, (p, q, r *distinct primes*) *is simple, then* $g = 60$ *or* $g = 168$. Using different methods, Burnside has treated such groups for $m = 4$ and odd g, and W. K. Turkin[3] for any odd g.

The order of a transitive permutation group $\mathfrak{G}$ of degree p is of the form $g = qp(1 + np)$ where q divides $p - 1$. These groups are the subject of a number of papers by Mathieu, Jordan, Sylow, Frobenius, Burnside, Miller and others.[4] It is possible to restrict oneself to the case that $\mathfrak{G}$ is simple. The elements of order p commute only with their own powers. We can prove

III. *Let $\mathfrak{G}$ be a simple group of order* $g = qp(1 + np)$ *where* $q|p - 1$, *in which the elements of order* p *commute only with their own powers. If* $n = (2p + 7)/3$ *then either (1) $\mathfrak{G}$ is cyclic, or (2) $\mathfrak{G} \cong LF(2, p)$, or (3) p is a prime of the form $2^h \pm 1$, and $\mathfrak{G} \cong LF(2, 2^h)$.*

The degrees of the irreducible representations for which $f \not\equiv 0 \pmod{p}$ can be given explicitly for many larger values of n. The group $LF(3, 3)$, $p = 13$ is an example for the case $n = (2p + 7)/3$. For the proof we need not assume that $\mathfrak{G}$ is a permutation group of degree p.

There are numerous groups for which our method allows us to find the degrees of the irreducible representations.

2.[5] Let $\mathfrak{G}$ be a group of order $g = p^a g'$ where the prime p does not divide g'. Let $\mathfrak{T}_1, \mathfrak{T}_2, \ldots, \mathfrak{T}_l$ be the distinct (= non-similar) irreducible representations of $\mathfrak{G}$ with coefficients in an algebraic number field K. If $\mathfrak{p}$ is a prime ideal dividing p then the coördinates in the underlying spaces can be chosen so that all the group elements are represented by matrices with $\mathfrak{p}$-integral coefficients. We replace every coefficient by its residue class (mod $\mathfrak{p}$). This operation will generally be indicated by a bar. Every representation $\mathfrak{T}_\lambda$ goes over into a modular representation $\overline{\mathfrak{T}}_\lambda$ of $\mathfrak{G}$ with coefficients in some Galois field $\overline{K}$ of characteristic p. Each $\overline{\mathfrak{T}}_\lambda$ can therefore be split into the irreducible modular representations $\mathfrak{F}_1, \mathfrak{F}_2, \ldots, \mathfrak{F}_k$ of $\mathfrak{G}$ in K,

$$\overline{\mathfrak{T}}_\rho \longleftrightarrow \sum_{\sigma=1}^{k} d_{\rho\sigma}\mathfrak{F}_\sigma. \tag{1}$$

If K is suitably chosen, the $\mathfrak{T}_\rho$ and $\mathfrak{F}_\sigma$ are absolutely irreducible. The Cartan invariants of the modular group ring of $\mathfrak{G}$ then are given by

$$c_{\kappa\lambda} = \sum_{\rho=1}^{l} d_{\rho\kappa}d_{\rho\lambda} \qquad (\kappa, \lambda = 1, 2, \ldots, k). \tag{2}$$

We may assume that K contains the g'th roots of unity. Let G be a *regular element* of $\mathfrak{G}$, i.e., an element whose order is prime to p. Every modular representation $\mathfrak{F}$ of $\mathfrak{G}$ in K represents G by a matrix $\mathfrak{F}(G)$ whose characteristic roots are the residue classes of g'th roots of unity mod $\mathfrak{p}$. There is, however, a (1–1)-correspondence between the g'th roots of unity and their residue classes. We may, therefore, define the *character* $\omega(G)$ of $\mathfrak{F}$ as the sum of the roots of unity which correspond to the characteristic roots of $\mathfrak{F}(G)$. The character $\omega(G)$ is a complex number; it is defined

for regular elements only. Two modular representations split into the same irreducible constituents, if and only if they have the same character (for all regular G).

Let $\mathfrak{C}_1, \mathfrak{C}_2, \ldots, \mathfrak{C}_k$ be the classes of conjugate elements which consist of regular elements, let g_λ be the number of elements in $\mathfrak{C}_\lambda$. The inverse elements of the elements of $\mathfrak{C}_\lambda$ form again a class $\mathfrak{C}_{\lambda^*}$, $(\lambda = 1, 2, \ldots, k)$. We arrange the values $\omega_\lambda^{(\kappa)}$ of the character $\omega^{(\kappa)}$ of $\mathfrak{F}_\kappa$ for the elements of $\mathfrak{C}_\lambda$ in form of a matrix, $X = (\omega_\lambda^{(\kappa)})$, where κ is the row-index and λ the column-index. The place of the orthogonality relations of the ordinary group characters is taken in the modular theory by

$$X'CX = \left(\frac{g}{g_\kappa}\,\delta_{\kappa,\lambda^*}\right) \qquad (\kappa \text{ row-index, } \lambda \text{ column-index}) \qquad (3)$$

where $\delta_{\rho\sigma} = 0$ for $\rho \neq \sigma$ and $\delta_{\rho\rho} = 1$, and where $C = (c_{\kappa\lambda})$ is the matrix of the Cartan invariants.

The ordinary and at the same time the modular representations of $\mathfrak{G}$ are distributed into "blocks" $B_1, B_2, \ldots$.[6] The ordinary representations of B_τ contain only modular representations of B_τ in (1).

The results of the remaining part of §2 were obtained jointly by C. Nesbitt and the author, and the proofs will be published in a separate paper. Let $\mathfrak{T}_\kappa$ be an irreducible ordinary representation whose degree is divisible by the highest possible power p^a of p. Then the corresponding modular representation $\bar{\mathfrak{T}}_\kappa$ is also irreducible; it is an indecomposable constituent of the modular regular representation. The representation $\mathfrak{T}_\kappa$ for itself forms a block B_τ (*blocks of "highest" kind*).

On the other hand, we consider *blocks of the lowest kind*, i.e., blocks which contain at least one ordinary character whose degree is prime to p. It can be shown that the number of such blocks is equal to the number of those $\mathfrak{C}_\lambda$ for which $(g_\lambda, p) = 1$. That is to say, the order g/g_λ of the normalizor of an element of $\mathfrak{C}_\lambda$ is divisible by p^a.

3. We say that two ordinary irreducible characters are p-conjugate if they are obtained from each other by a change in the choice of the primitive p^ath root of unity. Such characters have the same value for regular elements and consist therefore of the same modular constituents. This shows that they belong to the same block B_τ. Assume now that the degree t_ρ of an irreducible ordinary representation $\mathfrak{T}_\rho$ is divisible by p^{a-1}, that on the other hand the number r_ρ of distinct p-conjugate representations for $\mathfrak{T}_\rho$ is not divisible by p. Then it can be shown that $\mathfrak{T}_\rho$ contains each of its modular irreducible constituents only with the multiplicity 1, $d_{\rho\sigma} = 0$ or 1 in (1). If a second $\mathfrak{T}_\mu$ of degree t_μ is not p-conjugate to $\mathfrak{T}_\rho$, and if again the number r_μ of p-conjugates is prime to p, then $\mathfrak{T}_\rho$ and $\mathfrak{T}_\mu$ can only have a modular constituent $\mathfrak{F}$ in common, if

$$r_\rho t_\mu + r_\mu t_\rho \equiv 0 \pmod{p^a}. \qquad (4)$$

It follows that $t_\mu \equiv 0 \pmod{p^{a-1}}$. Further, in the case of an odd p, it follows easily from (4) and §2 that $\mathfrak{T}_\rho$ and $\mathfrak{T}_\mu$ and their p-conjugates are the only representations which contain $\mathfrak{F}$ as a modular constituent, and for which the number of p-conjugates is $\not\equiv 0 \pmod{p}$.

If $a = 1$, then $r_\rho \mid p - 1$, and hence always $(r_\rho, p) = 1$. Our results can here be applied to any representation $\mathfrak{T}_\rho$ of a block of lowest kind, since the assumptions concerning t_ρ, r_ρ are always satisfied. Using these facts, we can set up all the linear relations which are satisfied by the ordinary group characters, when we restrict ourselves to regular elements. This is the starting point for the considerations of §4 in which new properties of the group characters in the case $a = 1$ will be given.

For the actual determination of the numbers r_ρ of p-conjugate characters we need another fact which holds for any a and also for $p = 2$. *The determinant of* C *in* (3) *is always a power of* p, more exactly the highest power of p which divides $g^k/(g_1 g_2 \ldots g_k)$. This can be shown by proving that the other factors of the determinant on the right side in (3) are absorbed by the determinants $|X|$, $|X'| = |X|$ on the left side.

It follows that the elementary divisors of C are the powers of p which divide $g/g_1, g/g_2, \ldots, g/g_k$. If a block B_r contains at least one representation $\mathfrak{T}_\rho$ of a degree $\not\equiv 0 \pmod{p^\alpha}$, then at least one elementary divisor of the part of C, which corresponds to B_r, is $\geqq p^{a-\alpha}$. We thus obtain new conditions for the blocks of different types.

I mention some further results which hold for representations $\mathfrak{T}_\rho$ of a degree $t_\rho \equiv 0 \pmod{p^{a-1}}$ in the case where $r_\rho = 1$. Here, the arrangement of the modular constituents $\mathfrak{F}_\lambda$ of $\mathfrak{T}_\rho$ is uniquely determined apart from a cyclic permutation. If we have h such modular constituents in $\overline{\mathfrak{T}}_\rho$, then we can find h representations $\mathfrak{T}_\rho^{(1)}, \mathfrak{T}_\rho^{(2)}, \ldots, \mathfrak{T}_\rho^{(h)}$ which represent $\mathfrak{G}$ by matrices with $\mathfrak{p}$-integral coefficients such that all the $\mathfrak{T}_\rho^{(\nu)}$ are similar to $\mathfrak{T}_\rho$ but no two of them can be transformed into each other by means of a similarity transformation with $\mathfrak{p}$-integral coefficients with a determinant prime to $\mathfrak{p}$. They represent all the subclasses into which the class of all representations similar to $\mathfrak{T}_\rho$ splits, if ordinary similarity is replaced by similarity in this narrower sense.

4. From now on, we restrict ourselves to the case $a = 1$, p odd.

THEOREM: *Let* $\mathfrak{G}$ *be a group of order* $g = pg'$ *where* p *is an odd prime and* $g' \not\equiv 0 \pmod{p}$. *Denote by* $\mathfrak{N}$ *the normalizor of a Sylow-subgroup* $\mathfrak{P}$ *of order* p. *If the degree* t *of an ordinary irreducible character* χ *of* $\mathfrak{G}$ *is divisible by* p, *then* χ *vanishes for all elements of an order* $\equiv 0 \pmod{p}$. *For the characters* χ *of a degree* $t \not\equiv 0 \pmod{p}$, *we can set up a* $(1-1)$*-correspondence with the irreducible characters* ψ *of* $\mathfrak{N}$ *such that if* χ *corresponds to* ψ; *we have*

$$\epsilon(\chi)\cdot\chi(G) = \psi(G), \quad \epsilon(\chi) = \pm 1 \tag{5}$$

for elements G *of an order divisible by* p. *The sign* $\epsilon(\chi)$ *is independent of* G. *In particular, the number of characters* χ *with* $t \not\equiv 0 \pmod{p}$ *is equal to the number of classes of conjugate elements of* $\mathfrak{N}$.

The normalizor $\mathfrak{N}'$ of an element P of order p of $\mathfrak{P}$ is contained in $\mathfrak{N}$ as a normal subgroup. On the other hand, $\mathfrak{N}' = \mathfrak{M} \times \mathfrak{P}$, where $\mathfrak{M}$ has an order prime to p. If the irreducible representations of $\mathfrak{M}$ are known, then the characters of $\mathfrak{N}$ can be obtained without difficulty, and we can therefore write down the values of the characters of $\mathfrak{G}$ for all elements of an order $\equiv 0 \pmod{p}$, only the sign of the characters remaining undetermined.

For an element P of order p, we have

$$t \equiv \chi(P) \pmod{\mathfrak{p}}. \tag{6}$$

If therefore the degree of the character of $\mathfrak{G}$ is known, this sign can also be found.

On the other hand, if the character ψ of $\mathfrak{N}$ has the degree h, it follows from (5) and (6) that the degree t of the corresponding representation $\mathfrak{T}$ of $\mathfrak{G}$ satisfies

$$t \equiv \pm h \pmod{p}. \tag{7}$$

Combining this with the known properties of the degrees and with the formulas (9) below, we can actually find the degrees t for numerous groups even of very high orders.

The structure of $\mathfrak{M}$ is very simple in many important cases. Often, it is of order 1.

We now study the distribution of the ordinary characters of a degree $t \not\equiv 0 \pmod{p}$ into blocks. Suppose that $B_1, B_2, \ldots, B_s$ are the blocks consisting of such representations. All the following blocks consist of exactly one character of a degree $t \equiv 0 \pmod{p}$ (§2). There are exactly s classes of conjugate elements, say $\mathfrak{C}_1, \mathfrak{C}_2, \ldots, \mathfrak{C}_s$, such that $\mathfrak{C}_\sigma$ consists of regular elements and the number g_σ of its elements is $\not\equiv 0 \pmod{p}$ (§2). In each $\mathfrak{C}_\sigma$ ($\sigma = 1, 2, \ldots, s$) we can find an element V_σ which commutes with the element P of order p, we may take $V_1 = 1$. Let z_σ be the number of classes of conjugate elements which contain an element $V_\sigma P^\alpha$ with $\alpha \not\equiv 0 \pmod{p}$ for a fixed σ. If $n(G)$ denotes the order of the normalizor of the group element G, then z_σ can also be characterized by the conditions

$$z_\sigma | p - 1, \qquad n(V_\sigma) z_\sigma + n(V_\sigma P) \equiv 0 \pmod{p^2}.$$

We can show that the blocks $B_1, B_2, \ldots, B_s$ can be arranged so that B_λ *consists (1) of* $\dfrac{p-1}{z_\lambda}$ *characters, which lie in the field of* g'*th roots of unity, and (2) of* z_λ *further characters which are all* p*-conjugate to each other.* We denote one of the latter ones by $\chi^{(\lambda,0)}$ and the first ones by $\chi^{(\lambda,\mu)}$, $\mu = 1, 2,$

$\ldots, \frac{p-1}{z_\lambda}$. In the case $z_\lambda = 1$, any character of B_λ can be taken as $\chi^{(\lambda,0)}$. For $\mu \neq 0$, $\chi^{(\lambda,\mu)}(P)$ is either positive or negative, we set $\epsilon^{(\lambda,\mu)} = +1$ or $\epsilon^{(\lambda,\mu)} = -1$ accordingly. In the case $\mu = 0$, we define the sign $\epsilon^{(\lambda,0)}$ as the sign of $\sum_{\alpha=1}^{p-1} \chi^{(\lambda,0)}(P^\alpha)$. Using the results of §3 we can show that the characters of the blocks B_λ satisfy the relation

$$\sum_{\mu=0}^{p-1/z_\lambda} \epsilon^{(\lambda,\mu)} \chi^{(\lambda,\mu)}(G) = 0 \text{ for regular elements } G, \ (\epsilon^{(\lambda,\mu)} = \pm 1). \quad (8)$$

In particular, the degrees $l^{(\lambda,\mu)} = \chi^{(\lambda,\mu)}(1)$ of the $\chi^{(\lambda,\mu)}$ satisfy

$$\sum_{\mu=0}^{p-1/z_\lambda} \epsilon^{(\lambda,\mu)} l^{(\lambda,\mu)} = 0. \quad (9)$$

Finally, we mention a property of the characters $\chi^{(\lambda,\mu)}$ for nonregular elements. It can be shown that the expression

$$\epsilon^{(\lambda,\mu)} \chi^{(\lambda,\mu)}(V_\sigma P^\alpha) = \zeta_\sigma^{(\lambda)} \quad (10a)$$

has the same value for all $\mu > 0$ and all $\alpha \not\equiv 0 \pmod{p}$. For $\mu = 0$, the formula must be replaced by

$$\epsilon^{(\lambda,0)} \sum \chi^{(\lambda,0)}(V_\sigma P^\alpha) = \zeta_\sigma^{(\lambda)} \quad (10b)$$

where the sum on the left side is extended over all the z_λ distinct p-conjugates of $\chi^{(\lambda,0)}$. The expression $\zeta_\sigma^{(\lambda)}$ on the right side is the same as in (10a). There exists a character $\vartheta^{(\lambda)}$ of $\mathfrak{M}$, for which

$$\vartheta^{(\lambda)}(V_\sigma) = \zeta_\sigma^{(\lambda)} \quad (11)$$

and ϑ^λ is the sum of z_λ/z_1 irreducible associated characters of the subgroup $\mathfrak{M}$ of $\mathfrak{N}$. Each character of $\mathfrak{M}$ appears in exactly one $\vartheta^{(\lambda)}$. There is a close connection between these last facts and the theorem at the beginning of §4.

[1] *University of Toronto Studies, Mathematical Series* No. **4** (referred to under M. R. (1937)).

[2] *Finite Collineation Groups*, University of Chicago Press, 1917.

[3] W. Burnside, *Proc. Lond. Math. Soc.*, **33**, 266 (1901); W. K. Turkin, *Rec. Moscow*, **40**, 229 (1933).

[4] For references see E. Pascal, *Repertorium der höheren Mathematik*, Vol. **I**, part 1, pp. 211–214, Leipzig, 1910.

[5] For the results of §2, cf. M. R. For alternative definitions and proofs, cf. T. Nakayama, *Ann. Math.* (2) **39**, 361 (1938), and R. Brauer, these PROCEEDINGS, **25**, 252–258 (1939).

[6] Cf. M. R., pp. 17–21, and the last paper in [5].

ORDERED SETS, COMPLEXES AND THE PROBLEM OF COMPACTIFICATION

By J. W. Alexander

Institute for Advanced Study

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A collection β of sets B_i is said to have the *F. I. P.* (*finite intersection property*) if no finite subcollection of β has the intersection zero.[1]

LEMMA. *Let β be any collection of sets B_i with the F. I. P. Moreover, let each of the sets B_i be the union of a finite collection of sets A_{ij}, $B_i = U_j A_{ij}$. Then to each set B_i there may be made to correspond a set $A_{ij(i)}$ such that the collection α formed by the sets also has the $A_{ij(i)}$ F. I. P.*

For suppose we well-order the set β. Then we can show, without difficulty, that the first member B_1 of β can be replaced by a set $A_{1j(1)}$ so chosen that the resulting collection still has the F. I. P., that the second member B_2 can afterward be replaced by a set $A_{2j(2)}$ and so on. Moreover, we can also show, by transfinite induction, that the process can be continued until all the B_i's have been replaced by appropriate $A_{ij(i)}$'s.

A number of theorems about compact topological spaces follow directly from the lemma. For the sake of greater generality, we shall adopt the revised definition of a topological space S (in terms of points and *CB*-sets) suggested by the author in a recent note.[2] A set β of *CB*-sets of S will be said to be *basic* if every non-empty *CB*-set of S is the intersection of a subset of β; a set α will be said to be *sub-basic* if there exists a basic set β such that each member B_i of β is the union of a finite subset of α; $B_i = \Sigma_j A_{ij}$.

THEOREM 1. *Given a topological space S and a basic set β [sub-basic set α], a necessary and sufficient condition that S be boundedly compact is that no subset β' of β [α' of α] with the F. I. P. have the intersection zero.*

The necessity of the condition, for either formulation of the theorem, follows automatically from the definition of a boundedly compact space.[2] Its sufficiency is immediate for the formulation in terms of a basic set β, and is a simple consequence of the lemma for the formulation in terms of a sub-basic set α, since we can associate with α a basic set β such that $B_i = \Sigma_j A_{ij}$.

Now, consider an arbitrary (finite or infinite) collection of topological spaces S_i. Moreover, let P_i be a representative point and c_i a representative *CB*-set of S_i. Then the *product* S of the spaces S_i will be a space made up of one point P corresponding to every way of selecting a point P_i from each of the spaces S_i. There will be a sub-basic set of S made up of one *CB*-set c corresponding to every way of selecting a *CB*-set c_i from each of the spaces S_i. The point P will be a member of the set c if, and only if,

each "coördinate" P_i of P is a member of the corresponding "coördinate" c_i of c.

THEOREM 2. *The product S of an arbitrary collection of boundedly compact [compact] spaces S_i is boundedly compact [compact].*[3]

This is an immediate corollary of Theorem 1 formulated in terms of a sub-basic set α.

We now propose to broaden the definition of a topological complex in the following manner. Suppose we start with an arbitrary set of undefined entities x_i, called *vertices*. Then by a *simplex* we shall mean a perfectly arbitrary set of vertices (not necessarily a finite set as is ordinarily assumed). A simplex A will be a *face* of a simplex B if A is a subset of B. A *complex* Φ will be any set of simplexes such that if A is in the set then every face of A is also in the set. The finite simplexes of a complex Φ will form a sub-complex Φ_F of Φ called the *core* of Φ. A complex Φ will be *complete* provided every simplex A such that all its finite faces are in Φ is itself in Φ. Every complex Φ will thus admit a *completion* Φ_c consisting of all simplexes such that their finite faces are all in Φ. Obviously two complexes Φ and Ψ will have the same completion if, and only if, they have the same core.

A simplex A of Φ will be said to be *maximal* if it is not a face of any simplex B of Φ other than A itself.

THEOREM 3. *If Φ is a complete complex every simplex A of Φ is a face of at least one maximal simplex A^*.*

The simplex A^* can easily be constructed by the adjunction to A of a suitable finite or transfinite set of vertices.

To every complex Φ there corresponds an associated topological space S, obtained as follows. Let Φ_c be the completion of the complex Φ. Then, by definition, we shall call the maximal simplexes of Φ_c the points of S. Now, each vertex x will determine a point set c_x consisting of all points A^* such that x is in A^*. We shall complete the definition of the space S by saying that the point sets c_x form a sub-basic set of S.

THEOREM 4. *The topological space* S *associated with a complex Φ is boundedly compact.*

This is a simple consequence of Theorem 1 formulated in terms of a sub-basic set α. From the equivalence

$$x \in A^* \rightleftarrows A^* \in c_x$$

and from the completeness of Φ_c we readily show that every collection of sets c_x with the F. I. P. has a non-zero intersection.

A trivial extension of Theorem 4 is sometimes desirable. It may happen that among the vertices x there are certain vertices y playing a *preferred* rôle. If so, the complex Φ, in conjunction with the preferred vertices y, also determines a boundedly compact space S_y made up of the

same points as the space S but such that there is a sub-basic set of S_y consisting merely of the sets c_y determined by the preferred vertices y.

As an application of the last theorem and its extension, consider any (partially) ordered set σ made up of elements x, among which there may be certain preferred elements y. Then if we form the *nerve* Φ of the set σ,[4] this complex Φ, in combination with the preferred elements y, will determine a boundedly compact space S_y invariantively associated with σ. A case of particular interest is the one in which we start with a topological space T, denote by σ the set consisting of all points and of all *CB*-sets of T, and regard the *CB*-sets (but not the points) as the preferred elements of σ. Suppose we order the set σ by writing $a \leq b$ to denote that a is either a point or a subset of b. Then the space S_y invariantively associated with σ will be a boundedly compact extension of the original space T. We can map the space T on its extension S_y by identifying each point P of T with the point Q of S_y (i.e., the maximal simplex Q of Φ_c) consisting of P and of all *CB*-sets of T containing P. The mapping of T on S_y will be continuous, and will be dense in S_y. Finally, the closures in S_y of a finite set of *CB*-sets of T will have a common point if, and only if, the sets themselves have a common point. The space S_y will be completely determined by the above specifications. It will have the same dimensionality as T.[5]

[1] Cf. H. Wallman, "Lattices and Topological Spaces," *Ann. Math.*, **39**, 112–126 (1938).

[2] "On the Concept of a Topological Space," these PROCEEDINGS **25**, 52–54 (1939). On p. 52 of the note here referred to, line 16 of the text should be corrected to read: "(iii) The intersection of an arbitrary *non-empty* set," etc. At the suggestion of the local topologists, I shall hereafter discard the terms *bicompact* and *semi-bicompact* (loc. cit., p. 52, second paragraph and third footnote) in favor of *compact* and *boundedly compact*, respectively.

[3] This is essentially a theorem of A. Tychonoff "Ueber die topologische Erweiterung von Räumen," *Math. Ann.*, **102**, 544–561 (1930).

[4] As defined by Alexandroff, "Diskrete Räume," *Recueil Mathématique* **2**, 501–519 (1937). Each of the five complexes associated with an ordered set determines a boundedly compact space associated with the set.

[5] The above process is closely related to the one used by H. Wallman to compactify an arbitrary T_1-space of the classical type, loc. cit., first footnote. Lefschetz has recently shown that Wallman's process can be extended to an arbitrary classical space which need not satisfy any separation axiom. (Not yet published.) I have borrowed from Lefschetz the device used in the last paragraph of combining the points and *CB*-sets (in his case, closed sets) into a single system.

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